Mechanics of Solids

Stresses And Strains

Stresses And Strains (Part I)

UNITS

Force

1 kilo-Newton (kN) = 10^3 N

1 mega-Newton (MN) = 10^6 N

1 giga- Newton (GN) = 10^9 N

Stress

The stress is expressed in units of kN/m^2, MN/m^2, N/mm^2, Pascal(N/m^2).

Various relationships are

1 kgf = 9.8 N

1 tonne = 10 kN

1 kgf/cm^2 = 10^5 N/m^2 = 0.1 N/mm^2

1 N/mm^2 = 10^6 N/m^2 = 10^6 Pa = 1 MPa (Mega Pascal)

1 GPa = 10^3 MPa

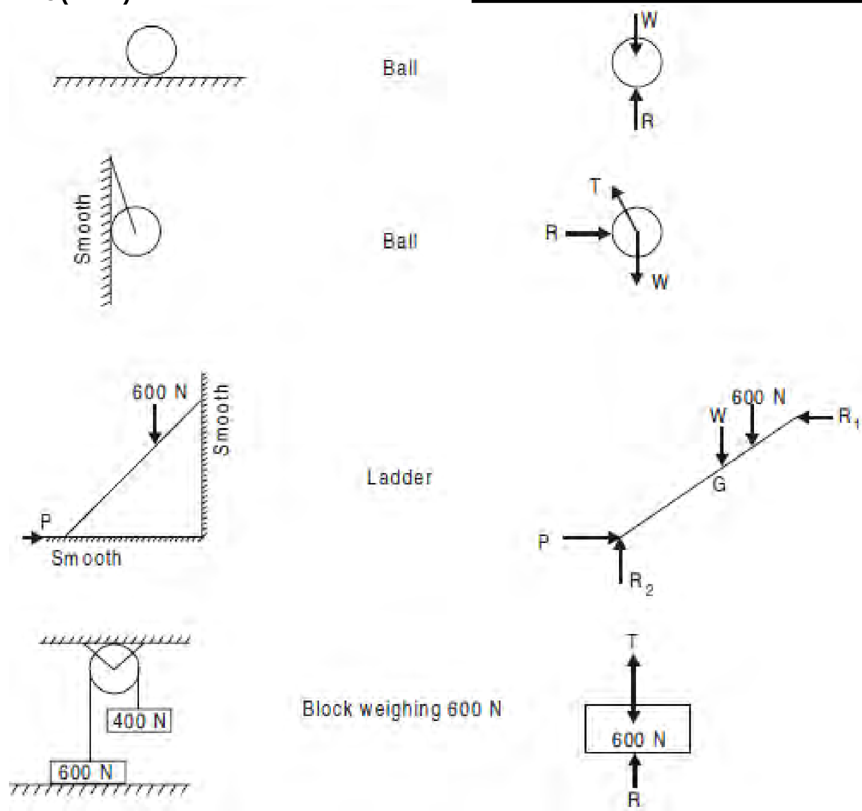
EQUILIBRIUM OF BODIES

Free Body Diagram (FBD)

The free body diagram of an element of a member in equilibrium is the diagram of only that member of element, as if made free from the rest, with all the external forces acting on it.

In many problems, it is essential to isolate the body under consideration from the other bodies in contact and draw all the forces acting on the body. For this, first the body is drawn and then applied forces, self weight and the reactions at the points of contact with other bodies are drawn. Such a diagram of the body in which the body under consideration is freed from all the contact surfaces and shows all the forces acting on it (including reactions at contact surfaces), is called a Free Body Diagram (FBD).

Following are a few FBD.



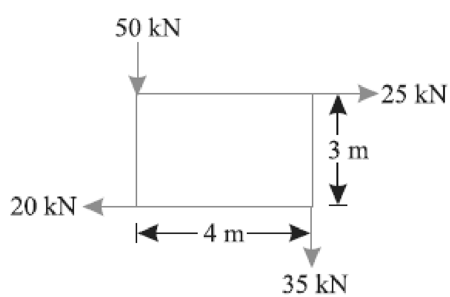
Body in Equilibrium

A body is said to be in equilibrium when it is at rest or has uniform motion. According to Newton's law of motion, it means the resultant of all the forces acting on a body in equilibrium is zero. The resultant of coplanar system of forces acting on a body is zero when

- The algebraic sum of the component of forces along each of the two mutually perpendicular directions is zero (translatory motion is zero). $\Sigma x = 0$ and $\Sigma y = 0$
- The algebraic sum of moment of all the forces about any point in the plane is zero (rotational moment is zero). $\Sigma M = 0$

Example

A system of forces are acting at the corners of a rectangular block as shown in following figure.



Solution

Given : System of forces

Magnitude of the resultant force

Resolving forces horizontally,

$$\Sigma H = 25 - 20 = 5 \text{ kN}$$

and now resolving the forces vertically

$$\Sigma V = (-50) + (-35) = -85 \text{ kN}$$

∴ Magnitude of the resultant force

$$R = (\Sigma H)^2 + (\Sigma V)^2 = (5)^2 + (-85)^2 = 85.15 \text{ kN}$$

Direction of the resultant force

Let θ = Angle which the resultant force makes with the horizontal.

We know that

$$\tan\theta = \frac{\sum V}{\sum H} = \frac{-85}{5} = -17$$

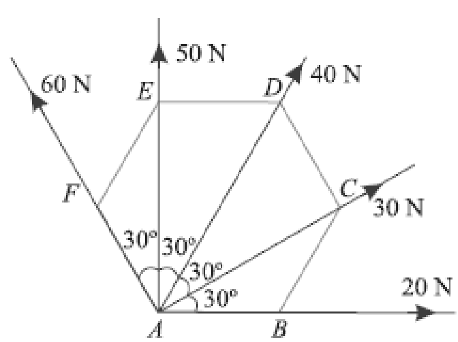
or $\theta = 86.6^0$

Since ΣH is positive and ΣV is negative, therefore resultant lies between 270° and 360°. Thus actual angle of the resultant force

$$= 360° - 86.6° = 273.4°$$

Problem

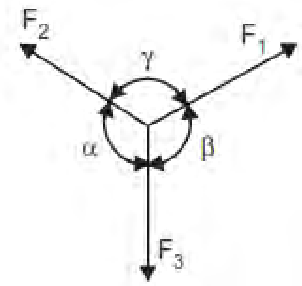
The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting at one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force.



Answer: 155.8 N; θ = 76.6^0

Lami's Theorem

If three forces acting at a point are in equilibrium, the ratio of any of the forces to the sine of the angle between the remaining two forces is the same.

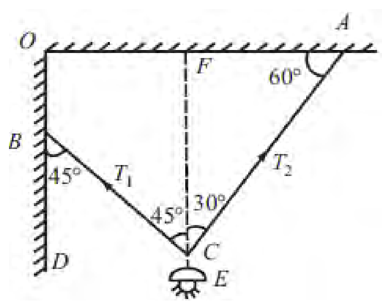


Let three forces F_1,F_2, and F_3 acting at a point be in equilibrium as shown in fig. above.

Then, $$\frac{F_1}{\sin\alpha}=\frac{F_2}{\sin\beta}=\frac{F_3}{\sin\gamma}$$

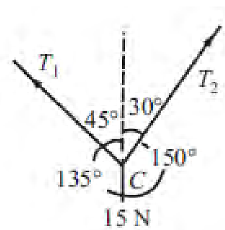
Example (AMIE W2007, 10 marks)

An Electric light fixture weighing 15N hangs from a point C, by two strings AC and BC. AC is inclined at 600 to the horizontal and BC at 450 to the vertical as shown in fig (6.54), Determine the forces in the strings AC and BC.



Solution

First draw the F.B.D. of the electric light fixture,



Apply Lami's theorem at point 'C'

$T_1/\sin 150° = T_2/\sin 135° = 15/\sin 75°$

$T_1 = 15.\sin 150°/ \sin 75°$

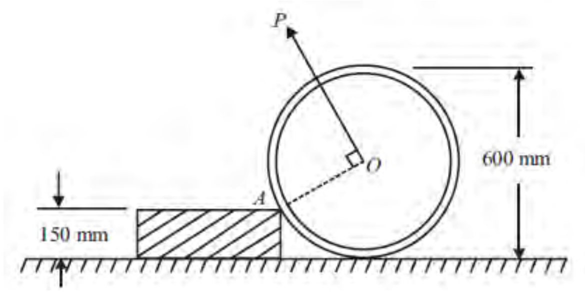
$\mathbf{T_1 = 7.76N}$

$T_2 = 15.\sin 135°/ \sin 75°$

$\mathbf{T_2 = 10.98N}$

Example

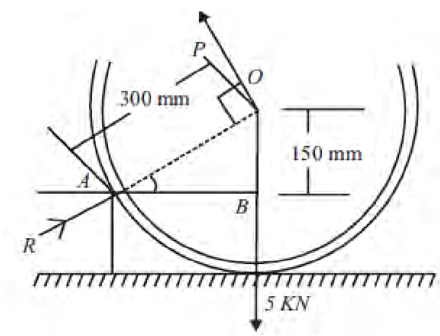
A uniform wheel of 600 mm diameter, weighing 5KN rests against a rigid rectangular block of 150mm height as shown in figure. Find the least pull, through the center of the wheel, required just to turn the wheel over the corner A of the block. Also find the reaction of the block. Take the entire surface to be smooth.



Solution

Let P = least pull required just to turn the wheel

Least pull must be applied normal to AO. F.B.D of wheel is shown in fig., from the fig.,



$\sin \theta = 150/300$, $\theta = 30°$

$AB = \{(300)^2 - (150)^2\}^{1/2} = 260$ mm

Now taking moment about point A, considering body is in equilibrium

$P \times 300 - 5 \times 260 = 0$

P = 4.33 KN

Calculation for reaction of the block

Let R = Reaction of the block

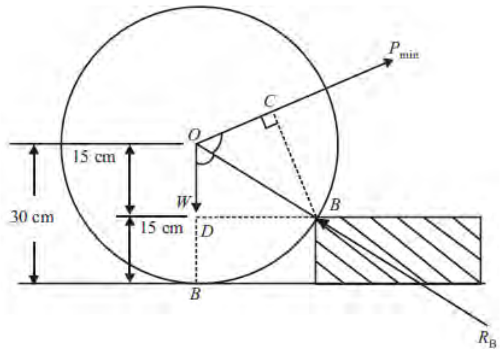
Since body is in equilibrium, resolving all the force in horizontal direction and equate to zero,

$R \cos 30° - P \sin 30° = 0$

R = 2.5KN

Example

A uniform wheel of 60 cm diameter weighing 1000 N rests against rectangular obstacle 15 cm high. Find the least force required which when acting through center of the wheel will just turn the wheel over the corner of the block. Find the angle of force with horizontal.



Solution

Let, P_{min} = Least force applied as shown in fig.

α = Angle of the least force

From triangle OBC,

$BC = BO\sin\alpha$

$BC = 30\sin\alpha$

In Triangle BOD, $BD = \{(BO)^2 - (OD)^2\}^{1/2}$

$BD = (30^2 - 15^2)^{1/2} = 25.98$

Taking moment of all forces about point B, We get

$P_{min} \times BC - W \times BD = 0$

$P_{min} - W \times BD/BC$

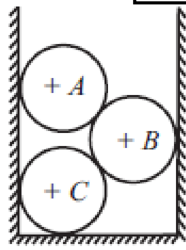
$P_{min} = 1000 \times 25.98 / 30\sin\alpha$

We get minimum value of P when α is maximum and maximum value of α is at 90º i.e. 1, putting $\sin\alpha = 1$

$\mathbf{P_{min} = 866.02N}$

Example

Three sphere A, B, C are placed in a groove shown in figure. The diameter of each sphere is 100mm. Sketch the free body diagram of B. Assume the weight of spheres A, B, C as 1KN, 2KN and 1KN respectively.

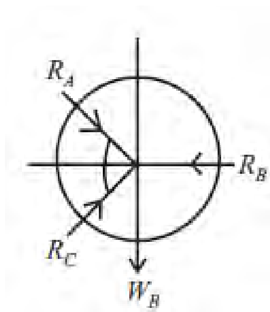


Solution

For θ,

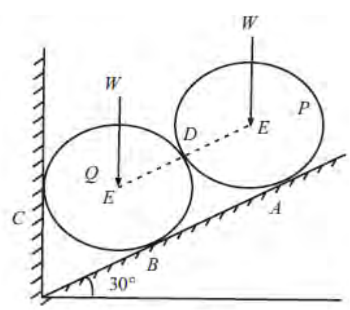
$$\cos\theta = 50/100,\ \cos\theta = .5\ ,\ \theta = 60°$$

FBD of block *B* is given in fig.



Example

Two cylindrical identical rollers A and B, each of weight W are supported by an inclined plane and vertical wall as shown in fig.



Assuming all surfaces to be smooth, draw free body

diagrams of

(i) roller A,

(ii) roller B

(iii) Roller A and B taken together.

Solution

Let us assumed

W = Weight of each roller

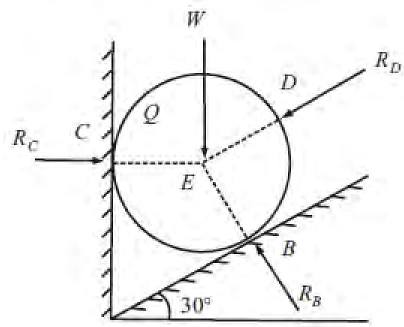
R = Radius of each roller

R_A = Reaction at point A

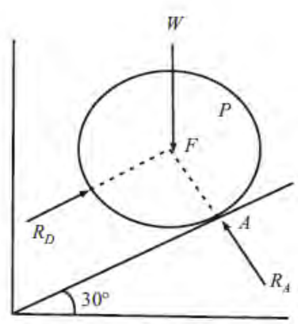
R_B = Reaction at point B

R_C = Reaction at point C

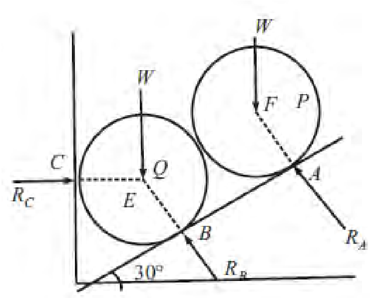
R_D = Reaction at point D



FBD of Roller 'B'



FBD of Roller 'A'

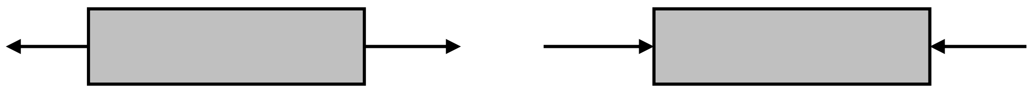


FBD of Roller 'B' & 'A' taken together

SIMPLE STRESSES & STRAINS

Axially Loaded Bar

The simplest case to consider at the start is that of an initially straight metal bar of constant cross section, loaded at its ends by a pair of oppositely directed collinear forces coinciding with the longitudinal axis of the bar and acting through the centroid of each cross section. For static equilibrium the magnitudes of the forces must be equal. If the forces are directed away from the bar, the bar is said to be in **tension**; if they are directed toward the bar, a state of **compression** exists. These two conditions are shown below.



Bar in tension Bar in compression

Normal Stress

The intensity of normal force per unit area is termed as normal stress and is expressed in units of force per unit area, e.g. N/m^2 (Pascal).If the forces applied to the ends of the bar are such that the bar is in tension, then resultant **tensile stresses** are set up in the bar; if the bar is in compression we have **compressive stresses**.

Normal Strain

Suppose a test piece is placed in tension. The elongation per unit length, which is termed as normal strain and denoted by a, may be found by dividing the total elongation Δ by the gauge length L, i.e. $\varepsilon = \Delta/L$. This is dimensionless.

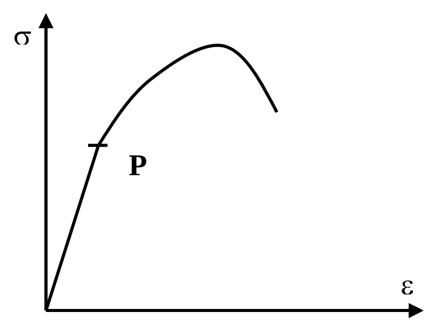
Stress Strain Curve

Normal Stress (σ) = P/A

where P is axial load in Newton and A is cross sectional area.

Having obtained various values of σ and ε, the experimental data may be plotted as below.

For alloy steel stress strain diagram is given below.



Hooke’s Law

For stress strain curve above, it is evident that the relation between stress and strain is linear for comparatively small values of the strain. This linear relation between elongation and the axial force causing it was first noticed by Sir Robert Hooke and is called Hooke’s law.

$\sigma = E\varepsilon$, where E denotes the slope of the straight-line portion OP in the curve.

Generalised Hook’s law to relate strain-stress for two dimensional state of stress is

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - v\sigma_y\right)$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - v\sigma_x\right)$$

where v is Poisson ratio.

For three dimensional state of stress, it is

$$\varepsilon_x = \frac{1}{E}\left[\left(\sigma_x - v\right)\left(\sigma_y + \sigma_z\right)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\left(\sigma_y - v\right)\left(\sigma_x + \sigma_z\right)\right]$$

$$\varepsilon_z = \frac{1}{E}\left[\left(\sigma_z - v\right)\left(\sigma_x + \sigma_y\right)\right]$$

Gauge Length

Gauge length is the failure length of the parallel portion of the test piece over which extensions are measured. For cylindrical test pieces, the gauge length used for calculation of percentage elongation is a function of the cross-sectional area of the diameter of the test bar.

Yield Point

Yield point is the point at which there is an appreciable elongation or yielding of material without any corresponding increase of load.

Yield Strength

The yield strength, closely associated with the yield point, is defined as the lowest stress at which extension of the test piece increases without increase in load.

Ultimate Strength

Ultimate stress or ultimate strength corresponds to the highest point of the stress strain curve.

Ultimate strength is maximum load/original cross sectional area.

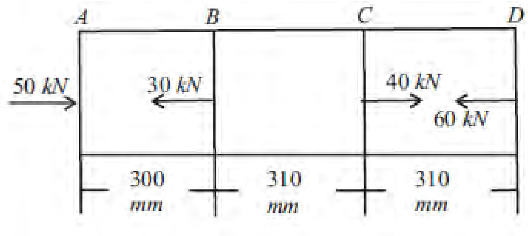
Percentage Elongation

The percentage elongation is the percentage increase in length of gauge length. If L_0 is the original gauge length and L_f is the final gauge length (measured after fracture) then

$$\text{Percentage elongation} = \frac{L_f - L_0}{L_0} \text{x} 100$$

Example

A steel bar is subjected to loads as shown in fig.. Determine the change in length of the bar ABCD of 18 cm diameter. E = 180 kN/mm^2.



Solution

Ref given figure.

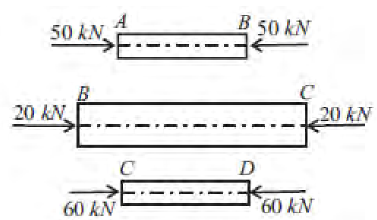
Since d = 180mm; E = 180 × 10^3 N/mm^2

L_{AB} = 300mm

L_{BC} = 310mm

L_{CD} = 310mm

From following figure



Load on portion AB = P_{AB} = 50 × 10^3 N

Load on portion BC = P_{BC} = 20 × 10^3 N

Load on portion CD = P_{CD} = 60 × 10^3 N

Area of portion AB = Area of portion BC

= Area of portion CD = A = $\pi d^2/4$

= $\pi(180)^2/4$ = 25446.9 mm^2

Using the relation $\delta l = Pl/AE$

$\delta L_{ab} = (50 \times 10^3 \times 300)/(25446.9 \times 180 \times 10^3) = 0.0033$mm -----Compression

$\delta L_{bc} = (20 \times 10^3 \times 310)/(25446.9 \times 180 \times 10^3) = 0.0012$mm -----Compression

$\delta L_{cd} = (60 \times 10^3 \times 310)/(25446.9 \times 180 \times 10^3) = 0.0041$mm -----Compression

Since net change in length = $-\delta L_{ab} - \delta L_{bc} - \delta L_{cd}$

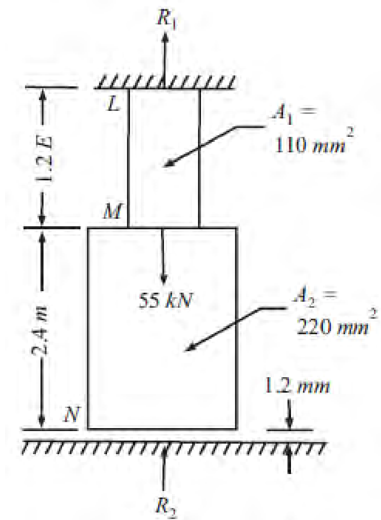
= – 0.0033 – 0.0012 – 0.0041

= – 0.00856 mm

Decrease in length = 0.00856mm

Example

For the bar shown in Fig., calculate the reaction produced by the lower support on the bar. Take E = 200 GN/m2. Find also the stresses in the bars.



Solution

Let R_1 = reaction at the upper support;

R_2 = reaction at the lower support when the bar touches it.

If the bar MN finally rests on the lower support,

we have

$R_1 + R_2 = 55kN = 55000$

N For bar LM, the total force = $R_1 = 55000 – R_2$ (tensile)

For bar MN, the total force =R_2 (compressive)

δL_1 =extension of LM = $[(55000 – R_2) \times 1.2]/[(110 \times 10 – 6) \times 200 \times 109]$

δL_2 = contraction of MN = $[R_2 \times 2.4]/[(220 \times 10–6) \times 200 \times 109]$

In order that N rests on the lower support, we have from compatibility equation

$\delta L_1 – \delta L_2 = 1.2/1000 = 0.0012$ m

Or, $[(55000 – R_2) \times 1.2]/[(110 \times 10–6) \times 200 \times 109] – [R_2 \times 2.4]/[(220 \times 10–6) \times 200 \times 109] = 0.0012$

on solving;

R_2 = 16500N or, 16.5 KN

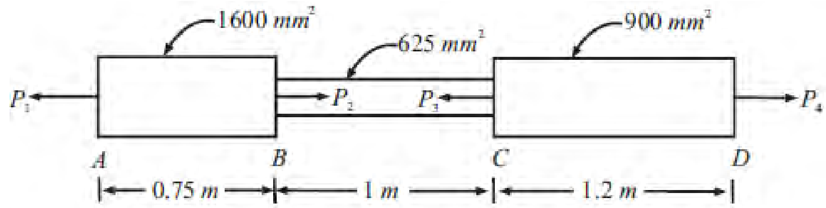
R_1 = 55-16.5 = 38.5 KN

Stress in LM = R_1/A_1 = 38.5/110 × 10–6 = 0.350 × 106 kN/m^2 = **350 MN/m^2**

Stress in MN = R_2/A_2 = 16.5/220 × 10–6 = 0.075 × 106 kN/m^2 **= 75 MN/m^2**

Example

A member ABCD is subjected to point loads P_1, P_2, P_3 and P_4 as shown in figure.



Calculate the force P_3 ,necessary for equilibrium if P_1 = 120 kN, P_2 = 220 kN and P_4 = 160 kN. Determine also the net change in length of the member. Take E = 200 GN/m^2.

Solution

Modulus of elasticity $E = 200\ GN/\text{m}^2 = 2 \times 10^5\ N/\text{mm}^2$.

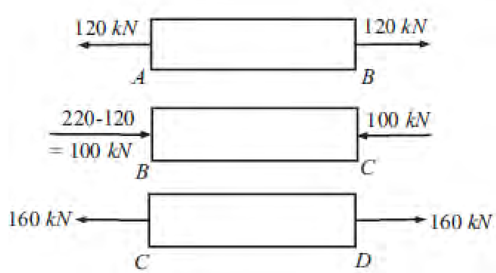
Considering equilibrium of forces along the axis of the member.

$$P_1 + P_3 = P_2 + P_4;$$

$$120 + P_3 = 220 + 160$$

Force $P_3 = 220 + 160 - 120 = 260$ kN

The forces acting on each segment of the member are shown in the free body diagrams shown below:



Let δL_1, δL_2 and δL_3, be the extensions in the parts 1, 2 and 3 of the steel bar respectively. Then,

Extension of segment AB

$$= [(120 \times 10^3) \times (0.75 \times 10^3)]/[1600 \times (2 \times 10^5)] = 0.28125 \text{ mm}$$

Compression of segment BC

$$= [(100 \times 10^3) \times (1 \times 10^3)]/[625 \times (2 \times 10^5)] = 0.8 \text{ mm}$$

Extension of segment CD

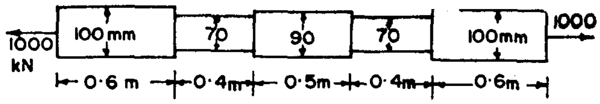
$$= [(160 \times 10^3) \times (1.2 \times 10^3)]/[900 \times (2 \times 10^5)] = 1.0667 \text{ mm}$$

Net change in length of the member

$$= \delta l = 0.28125 - 0.8 + 1.0667 = \mathbf{0.54795\ mm\ (increase)}$$

Problem

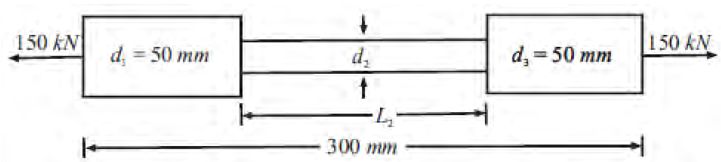
Find the total elongation of a steel bar as shown in following figure subjected to an axial load of 1000 kN. E = 200 GPa.



Answer : 2.196 mm

Example

The bar shown in figure is subjected to an axial pull of 150 kN. Determine diameter of the middle portion if stress there is limited to 125N/mm^2. Proceed to determine the length of this middle portion if total extension of the bar is specified as 0.15 mm. Take modulus of elasticity of bar material E = 2 × 10^5 N/mm^2.



Solution

Each of the segment of this composite bar is subjected to axial pull P =150 kN.

Axial Stress in the middle portion σ_2 = Axial pull/Area = $150 \times 10^3/[(\pi/4).(d_2^2)]$

$\therefore$ d_2 = 39.1 mm

Since stress is limited to 125 N/mm^2, in the middle portion

$$125 = 150 \times 10^3/[(\pi/4).(d_2^2)]$$

Diameter of middle portion

d_2 = 39.1mm

(ii) Stress in the end portions, $\sigma_1 = \sigma_3$

$$= 150 \times 10^3/[(\pi/4).(50^2)] = 76.43 \text{ N/m}^2$$

Total change in length of the bar,

= change in length of end portions + change in length of mid portion

$$\delta L = \delta L_1 + \delta L_2 + \delta L_3$$

$$= \sigma_1 L_1/E + \sigma_2 L_2/E + \sigma_3 L_3/E;$$ Since E is same for all portions

$$= \sigma_1(L_1 + L_3)/E + \sigma_2 L_2/E$$

$$L_1 + L_3 = 300 - L_2$$

Now putting all the values;

$0.15 = [76.43(300 – L_2)]/2 \times 10^5 + 125L_2/2 \times 10^5$

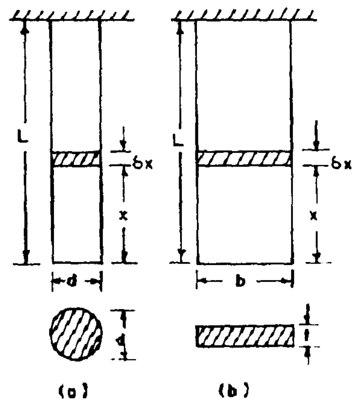
$\therefore$ **L_2 = 145.58 mm**

Example (AMIE Winter 2012, 8 marks)

A bar of uniform cross sectional area "A" and length "L" hangs vertically from a rigid support of the density of a material as ρ kg/m^3. Derive the expression for maximum stress induced and elongation produced in the bar due to its own weight.

Solution

See following figure



Consider a small section of length δx, at a distance x from the free end. The deformation $\delta\Delta$ is given by

$$\delta\Delta = \frac{W_s}{A}.\frac{\delta x}{E}$$

where W_s = weight of portion below the section = A.x.λ

$$\therefore \quad \delta\Delta = \frac{(Ax\lambda)\delta x}{A.E} = \frac{x\lambda\delta x}{E}$$

$\therefore$ Total deformation of the rod

$$\Delta = \int_0^L \frac{x\lambda}{E}dx = \frac{\rho g L^2}{2E}$$

Maximum stress induced in the strip

$$\sigma = \frac{\rho A x}{A} = \rho x$$

Example (AMIE S2006, 2012, Winter 2015, 10 marks)

Find the change in length of circular bar of uniform taper.

Solution

The stress at any cross section can be found by dividing the load by the area of cross section and extension can be found by integrating extensions of a small length over whole of the length of the bar. We shall consider the following cases of variable cross section:

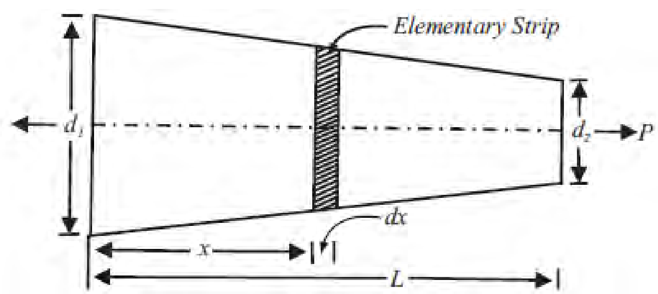
Consider a circular bar that tapers uniformly from diameter d1 at the bigger end to diameter d_2 at the smaller end, and subjected to axial tensile load P as shown in figure.

Let us consider a small strip of length dx at a distance x from the bigger end.

Diameter of the elementary strip:

$$dx = d_1 - [(d_1 - d_2)x]/L$$

$$= d_1 - kx \text{; where } k = (d_1 - d_2)/L$$



Cross sectional area of strip

$$A_s = \frac{\pi}{4} d_x^{\,2} = \frac{\pi}{4}(d_1 - kx)^2$$

Stress in the strip

$$\sigma_x = \frac{P}{A_x} = \frac{P}{\frac{\pi}{4}(c_1 - kx)^2} = \frac{4P}{\pi(d_1 - kx)^2}$$

Strain in strip

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{4P}{\pi(d_1 - kx)^2 E}$$

Elongation of the strip

$$\delta l_x = \varepsilon_x dx = \frac{4Pdx}{\pi(d_1 - kx)^2 E}$$

The total elongation of this tapering bar can be worked out by integrating the above expression between limits x = 0 to x = L.

$$\delta l = \int_0^L \frac{4Pdx}{\pi(d_1 - kx)^2 E} = \frac{4P}{\pi E}\int_0^L \frac{dx}{(d_1 - kx)^2}$$

$$= \frac{4P}{\pi E}\left[\frac{(d_1 - kx)^{-1}}{(-1)x(-k)}\right]_0^L = \frac{4P}{\pi EK}\left[\frac{1}{d_1 - kx}\right]_0^L$$

Putting the value of $k = (d_1 - d_2)/l$ in the above expression, we obtain

$$\delta l = \frac{4PL}{\pi E(d_1 - d_2)}\left[\frac{1}{d_1 - \frac{(d_1 - d_2)l}{l}} - \frac{1}{d_1}\right]$$

$$= \frac{4PL}{\pi E(d_1 - d_2)}\left[\frac{1}{d_1} - \frac{1}{d_1}\right] = \frac{4PL}{\pi E d_1 d_2}$$

Example

A steel rod, circular in cross section, tapers from 2.5 cm diameter to 1.25 cm diameter in a length of 50 cm. Find how much of this length will increase under a pull of 25 kN if E = 210 GPa.

Solution

Given: d_1 = 1.25 cm, L = 50 cm, d_2 = 2.5 cm, and P = 25 kN

Extension of a tapering circular bar

$$\Delta = \frac{4PL}{\pi d_1 d_2 E} = \frac{4 \text{ x } 25 \text{ x } 10^3 \text{ x } 0.5}{\pi \text{ x } 1.25 \text{ x } 2.5\text{x } 10^{-4} \text{ x } 210 \text{ x } 10^9} = 0.2425 \text{ mm.}$$

Example (AMIE Summer 2000, 2009, Winter 2006, 2012, 10 marks)

A tension bar is found to taper uniformly from (D - a) cm diameter to (D + a) cm. Prove that the error involved in using the mean diameter to calculate Young's Modulus is $\left\{\frac{10a}{D}\right\}^2$ *per cent.*

Solution

Given: d_1 = D-a, d_2 = D + a

Let L = Length of the bar and P = Load applied.

Mean diameter

$$D = \frac{d_1 + d_2}{2}$$

Mean stress $\sigma = \frac{4P}{\pi D}$

If u = Extension of the bar

then strain $\varepsilon = u/L$

Young modulus

$$E = \frac{4Pl}{4D^2u}$$

Now for a tapering round bar

$$u = \frac{4PL}{\pi d_1 d_2 E}$$

$$E = \frac{4PL}{\pi d_1 d_2 u}$$

$$\therefore \quad E = \frac{4PL}{\pi(D-a)(D+a)u} = \frac{4PL}{\pi(D^2-a^2)u}$$

$\therefore$ Error in Young's Modulus will be

$$\frac{4PL}{\pi u}\left\{\frac{1}{D^2-a^2} - \frac{1}{D^2}\right\} = \frac{4PL}{\pi u}\left\{\frac{a^2}{D^2(D^2-a^2}\right\}$$

$\therefore$ Percentage error will be

$$\frac{4PL}{\pi u}\left\{\frac{a^2}{D^2(D^2-a^2)}\right\} \text{ x } \frac{\pi u(D^2-a^2)}{4PL} \text{ x } 100 = \frac{a^2}{D^2} \text{ x } 100 = \left\{\frac{10a}{D}\right\}^2$$

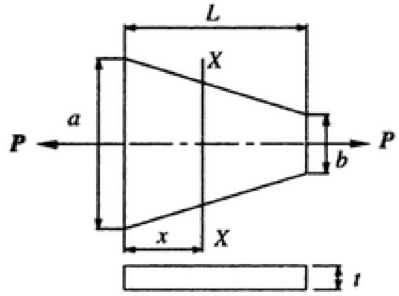
Hence Proved.

Example (AMIE S16, 8 marks)

A bar of steel of length "l" and is of uniform thickness ‘"t" The width of the bar varies uniformly from a at one end to b (a > b) at the other end. Find the extension of the rod when it carries an axial pull p.

Solution

Figure



Consider any section X-X distance x from the bigger end.

Area of section

Width of section

$$= a - \frac{a-b}{L}x$$

$$= a - kx \quad where\, k = \frac{a-b}{2}$$

Thickness of section = t

Area of section = t(a – kx)

Stress

Stress on the section

$$\delta x = \frac{P}{t(a-kx)}$$

Extension

Extension of an elemental length

$$\delta x = \frac{Pdx}{t(a-kx)E}$$

Total extension of the rod

$$\delta L = \frac{P}{tE}\int_0^L \frac{\delta x}{(a-kx)} = -\frac{P}{tEk}[\log_e(a-kL) - \log_e a$$

$$= \frac{P}{tEk}\log\left[\frac{a}{a-kL}\right]$$

Example (AMIE Winter 2007, 10 marks)

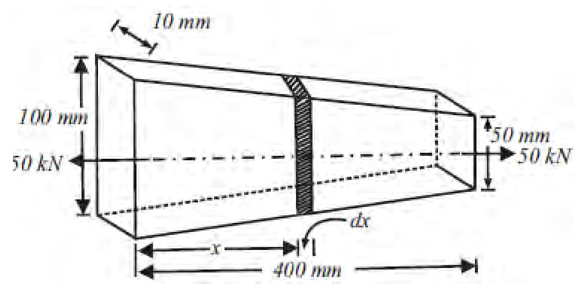
A steel bar AB of uniform thickness 2 cm, tapers uniformly from 100 mm to 50 mm in a length of 400 mm. From first principles determine the elongation of plate; if an, axial tensile force of 50 kN is applied on it. [E = 2 x 10^5 N/mm^2]

Solution

Consider a small element of length dx of the plate, at a distance x from the larger end. Then at this section,

Width $W_x = 100-(100-50)(x/400) = \left(100-\frac{x}{8}\right) mm$

Cross section area A_x = thickness x width = $10\left(100-\frac{x}{8}\right) mm^2$



Stress $$\sigma_s = \frac{P}{A_x} = \frac{50x10^3}{10(100-x/8)} = N/mm^2$$

Strain $$\varepsilon_x = \frac{\sigma_x}{E} = \frac{5x10^3}{(100-x/8)(2x10^5)} = \frac{1}{40(100-x/8)}$$

$\therefore$ Elongation of the elementary length

$$\delta L_x = \varepsilon_x x dx = \frac{dx}{40(100-x/8)}$$

The total change in length of the plate can be worked out by integrating the above identity between the limits x = 0 and x = 400 mm.

i.e. $$\delta L = \int_0^{400} \frac{dx}{40(100-x/8)} = \frac{1}{40}\int_0^{400}\frac{dx}{(100-x/8)} = 0.1386 \text{ mm}$$

Problem

A flat steel plate is of trapezoidal form. The thickness of the plate is 15 mm and it tapers uniformly from a width of 60 mm to 10 mm in a length of 30 cm. If an axial force of 100 kN is applied at each end, determine the elongation of the plate. Take E = 204 kN/mm^2.

Answer : 0.351 mm

Problem

A steel bar AB of uniform thickness 15 mm, tapers uniformly from 60 mm to 10 mm in a length of 300 mm. From first principles determine the elongation of plate; if an, axial tensile force of 120 kN is applied on it. [E = 2.04 x 10^5 N/mm^2]

Answer: $\Delta = 0.422$ *mm*

Example

A metal bar 5 cm x 5 cm section is subjected to an axial compressive load of 500 kN. The contraction on a 20 cm gauge length is found to be 0.5 mm and the increase in thickness 0.045 mm. Find the value of Young's modulus and Poisson's ratio.

Solution

Given P = 500 kN, l=20 cm, $\Delta l = 0.05$ cm, $\Delta t = 0.0045$ cm.

Area of cross section A = 5 x 5 = 25 cm^2

Longitudinal strain

$$\varepsilon = \Delta l/l = 0.05/20 = 0.0025 \text{ (compressive)}$$

stress $\sigma = P/A = 500 \times 10^3/25 \times 10^{-4} = 200$ MPa (compressive)

Young's Modulus

$$E = \sigma/\varepsilon = 200 \times 10^6/0.0025 = 80 \text{ GPa}$$

Lateral strain

$$\Delta t/t = 0.0045/5 = 0.0009 \text{ (tensile)}$$

Poisson's Ratio

$$v = \text{Lateral strain/Longitudinal} = 0.0009/0.0025 = 0.36$$

Example

A 70 cm length of aluminium alloy bar is suspended from the ceiling so as to provide a clearance of 0.03 cm between it and a 25 cm length of steel as shown in following figure.

$A_{al} = 12.5$ cm^2, $E_{al} = 70$ GN/m^2; $A_s = 25$ cm^2 $E_s = 210$ GN/m^2

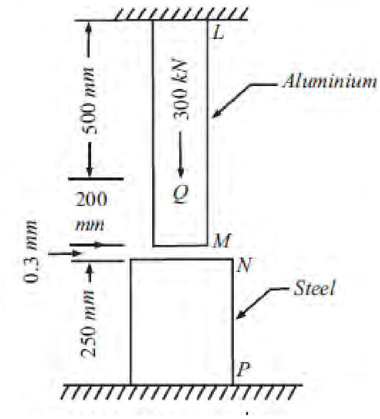
Determine the stress in the aluminium and in the steel due to a 300 kN load applied 50 cm from the ceiling.

Solution

Elongation of AB under 300 kN load will be

$$\frac{300 \times 10^3 \times 50 \times 10^{-2}}{12.5 \times 10^{-4} \times 70 \times 10^9} = 1.71428 \times 10^{-3} \text{ m} = 0.171428 \text{ cm.}$$

This elongation is more than the clearance 0.03 cm between the bars. Hence bars DE and BC will be subjected to compression whereas AB will remain in tension. Let P be compressive force is BC and DE then the tensile force in AB will be (300-P) kN. Then,



$$\frac{(300 - P) \times 10^3 \times 50 \times 10^{-2}}{12.5 \times 10^{-4} \times 70 \times 10^9} - \frac{P \times 10^3 \times 20 \times 10^{-2}}{12.5 \times 10^{-4} \times 70 \times 10^9} - \frac{P \times 10^3 \times 25 \times 10^{-2}}{25 \times 10^{-4} \times 210 \times 10^9}$$

$$= 0.03 \times 10^{-2}$$

$\therefore$ P = 166.854 kN

$$\sigma_{AB} = \frac{(300 - 166.854) \times 10^3}{12.5 \times 10^{-4}} = 106.52 \text{ MN/m}^2 \text{ (tensile)}$$

$$\sigma_{BC} = \frac{166.854 \times 10^3}{12.5 \times 10^{-4}} = 133.48 \text{ MN/m}^2 \text{ (compressive)}$$

$$\sigma_{DE} = \frac{166.854 \times 10^3}{25 \times 10^{-4}} = 66.74 \text{ Mn/m}^2 \text{ (compressive)}$$

Example

A rigid bar AB is hinged at A and supported by a bronze rod 2 m long and a steel rod 1 m long. A load of 80 tonnes is applied at the end B as shown in following figure. Using the data in the following table, calculate the stress in each rod and reaction at A.

	Area(mm^2)	*E(N/mm^2)*
Steel	*900*	*2.05×10^5*
Bronze	*600*	*0.82×10^5*

Solution

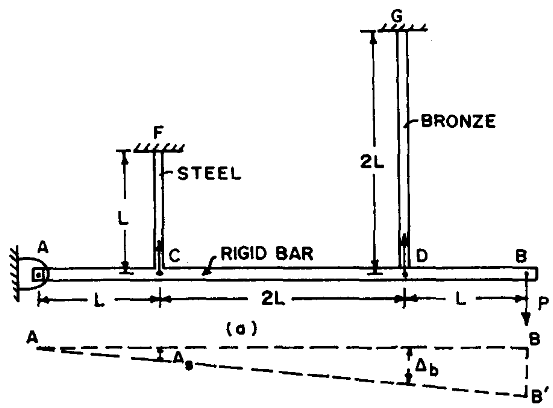
There are three unknowns, i.e. reaction R_A at A, force P_1 in the steel rod and force P_2 in the bronze rod. To solve the problem, the two equations of static equilibrium(i.e. the force equation and moment equation) must be used in combination with the additional equations obtained from the geometric relations between the elastic deformations.

Let suffix *s* stand for the steel rod and *b* for the bronze rod.

Taking moments about A, we get

$$\Sigma M_A = 0 = (P_1 \text{ x } 1) + (P_2 \text{ x } 3) - (80000 \text{ x } 4)$$

or $$P_1 + 3P_2 = 320000 \quad (1)$$



If Δ_1 is the deformation of the steel rod and Δ_2 that of bronze rod we have from similar triangles,

$$\frac{\Delta_s}{1} = \frac{\Delta_b}{3}$$

$\therefore$ $$\frac{P_1 L_1}{A_1 E_1} = \frac{1}{3}\frac{P_2 L_2}{A_2 E_2}$$

or $$\frac{P_1 \text{ x } 1000}{900 \text{ x } 2.05 \text{ x } 10^5} = \frac{1}{3}\frac{P_2 \text{ x } 2000}{600 \text{ x } 0.82 \text{ x } 10^5}$$

from which $$P_1 = 2.5\, P_2 \quad (2)$$

Substituting the value of P_1 in (1), we get

$$2.5P_2 + 3P_2 = 320000$$

From which, $\mathbf{P_2 = 58182\ N}$

$\mathbf{P_1 = 145455\ N}$

For finding the reactions at A, use the force equation(i.e. Σ P = 0). Assuming R_A to act downward,

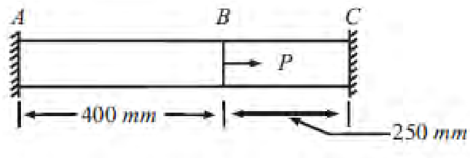
$$P_1 + P_2 - R_A - 80000 = 0$$

$\therefore$ $R_A = 145455 + 58182 - 80000 = 123637$ N **Answer**

(The positive sign to R_A shows that the assumed direction of R_A is correct.)

Example

A square bar of 25 mm side is held between two rigid plates and loaded by an axial pull equal to 300 kN as shown in figure. Determine the reactions at end A and C and elongation of the portion AB. Take E = 2 × 10⁵ N/mm².



Solution

Cross section area of the bar A = 25 × 25 mm^2

Since the bar is held between rigid support at the ends, the following observations need to be made:

- Portion AB will be subjected to tension and portion BC will be under compression
- Since each ends are fixed and rigid and therefore total Elongation;

  $\delta L_{ab} - \delta L_{bc} = 0$; Elongation in portion AB equals shortening in portion BC. i.e., $\delta L_{ab} = \delta L_{bc}$

- Sum of reactions equals the applied axial pull i.e., $P = R_a + R_c$

Apply second condition, we get

$$[P_{ab} \times L_{ab}]/A_{ab}.E = [P_{bc} \times L_{bc}]/A_{bc}.E$$

$$(R_a \times 400)/(625 \times 2 \times 10^5) = (R_c \times 250)/(625 \times 2 \times 10^5)$$

$$R_c = 1.6R_a \qquad \text{(i)}$$

Now apply third condition i.e., $P = R_a + R_c$

$$300 \times 10^3 = R_a + 1.6\, R_a$$

$$\mathbf{R_a = 1.154 \times 10^5\ N;\ R_c = 1.846 \times 10^5\ N}$$

ELASTIC CONSTANTS

Modulus of Elasticity (E)

The quantity E, i.e. ratio of the unit stress to the unit strain, is the modulus of elasticity of the material in tension, or, as it is often called Young's modulus.

Bulk modulus (K)

It is defined as the ratio of uniform stress intensity to volumetric strain, within the elastic limits and is denoted by K.

Poisson's Ratio

When a bar is subject to a simple tensile loading there is an increase in length of the bar in the direction of the load, but a decrease in the lateral dimensions perpendicular to the load. The ratio of the strain in the lateral direction to that in the axial direction is defined as Poisson's ratio. It is generally denoted by μ or 1/m. Its values lies between 0.25 and 0.35.

Modulus of rigidity (G)[1]

It is defined as the ratio of shearing stress to shearing strain.

Volumetric Strain

When a member is subjected to stresses, it undergoes deformation in all directions. Hence, there will be change in volume. The ratio of the change in volume to original volume is called volumetric strain.

Thus $$e_v = \frac{\delta V}{V}$$

where e_v = Volumetric strain

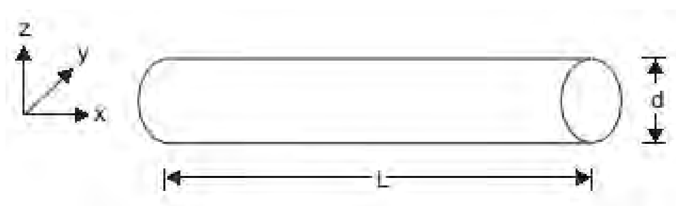
δV = Change in volume

V = Original volume

It can be shown that volumetric strain is sum of strains in three mutually perpendicular directions.

i.e., $$e_v = e_x + e_y + e_z$$

Let us consider a circular rod of length L and diameter 'd' as shown in figure.



Now $$V = \frac{\pi}{4} d^2 L$$

$$\delta V = \frac{\pi}{4} 2d\delta dL + \frac{\pi}{4} d^2 \delta L$$

$$\therefore \quad \frac{\delta V}{\frac{\pi}{4} d^2 L} = 2\frac{\delta d}{d} + \frac{\delta L}{L}$$

$$e_v = e_x = e_z = \frac{\delta d}{d}$$

[1] In some books it is represented by N.

In general for any shape volumetric strain may be taken as sum of strains in three mutually perpendicular directions.

Relation between E and K

$$E = 3K\left(1 - \frac{2}{m}\right)$$

Proof: Let a cube of side L be subjected to three mutually perpendicular like stresses of equal intensity q. By the definition of bulk density

$$\frac{p}{e_v} = K$$

or $$e_v = \frac{\delta V}{V} = \frac{p}{K} \qquad \text{(i)}$$

The total linear strain of each side

$$e = \frac{p}{E} - \frac{p}{mE} - \frac{p}{mE}$$

∴ $$\frac{\delta L}{L} = e = \frac{p}{E}\left(1 - \frac{2}{m}\right) \qquad \text{(ii)}$$

Now $V = L^3$

∴ $\delta V = 3L^2\delta L$

or $$\frac{\delta V}{V} = e_v = \frac{3\delta L}{L} = 3e = \frac{3p}{E}\left(1 - \frac{2}{m}\right) \qquad \text{(iii)}$$

Equating (i) and (iii)

$$\frac{p}{K} = \frac{3p}{E}\left(1 - \frac{2}{m}\right)$$

or $$E = 3K\left(1 - \frac{2}{m}\right)$$

Relation between E and G

$$E = \frac{9KG}{G + 3K}$$

Proof. Let us consider a square block ABCD, subjected to a state of simple shear q. There will be some distortion.

Linear strain e of the diagonal AC = q/2G. (i)

Now, linear strain e of the diagonal AC, due to these two mutually perpendicular direct stresses, is given by

$$e = \frac{p}{E} - \left(-\frac{p}{mE}\right) = \frac{p}{E}\left(1 + \frac{1}{m}\right) \qquad \text{(ii)}$$

Equating (i) and (ii)

$$\frac{q}{2G} = \frac{q}{E}\left(1 + \frac{1}{m}\right)$$

or $$E = 2G\left(1 + \frac{1}{m}\right)$$

which gives relation between E and G.

We have already proved that

$$E = 3K\left(1 - \frac{2}{m}\right)$$

Equating the two, we get

$$E = 2G\left(1 + \frac{1}{m}\right) = 3K\left[1 - \frac{2}{m}\right]$$

Eliminating E, we get

$$\frac{1}{m} = \frac{3K - 2N}{6K + 2N}$$

Eliminating m, we get

$$E = \frac{9KN}{N + 3K}$$

Example (AMIE Winter 2010, 8 marks)

There is a 2% error in the determination of G. If E is assumed to be correctly determined, what will be the error in the calculation of Poisson's ratio when its correct value is 0.2?

Solution

We know $E = 2G(1+m)$

Applying the principle of differential calculus, we can differentiate above equation keeping in mind that δE = 0, because E is constant.

∴ $$0 = 2\delta G(1+m) + 2G\delta v$$

or $$\delta v = -\frac{\delta G}{G}(1+m)$$

% error will be

$$\frac{\delta m}{m} x100 = -\frac{\delta G(1+m)}{m} x100 = -\frac{2}{100}\left(\frac{1+0.2}{0.2}\right) x100 = -12\%$$

Example

A steel bar of rectangular cross section 20 mm x 10 mm is subjected to a pull of 20 kN in the direction of its length. Taking E = 0.2 x 10^6 N/mm^2 and m = 10/3, find the length of the sides of the cross section and percentage decrease of area of cross section.

Solution

The strain in the direction of pull is

$$e_1 = \frac{p}{m} = \frac{P}{AE} = \frac{20000}{20 \times 10 \times 0.2 \times 10^6} = 5 \times 10^{-4}$$

Lateral strain = $-\frac{e_1}{m} = -\frac{3}{10} \times 5 \times 10^{-4}$

Hence 20 mm side is decreased by 20 x 1.5 x 10^{-4} = 0.0030 mm

10 mm side is decreased by 10 x 1.5 x 10^{-4} = 0.0015 mm

New area of cross section = (20 - 0.003)(10 - 0.0015) = (200 - 0.06)

$\therefore$ % decrease of area of cross section = $\frac{0.06}{200} \times 100 = 0.03$ **Answer**

Example

A piece of steel 200 mm long and 20 mm x 20 mm cross section is subjected to a tensile force of 40 kN in the direction of its length. Calculate the change in volume. Take 1/m = 0.3. E = 2.05 x 10^5 N/mm^2.

Solution

$$e_1 = \frac{40000}{(20 \times 20)(2.05 \times 10^5)} = 4.88 \times 10^{-4}$$

$$e_2 = e_3 = -\frac{e_1}{m} = -4.88 \times 0.3 \times 10^{-4} = -1.464 \times 10^{-4}$$

$$\delta V/V = e_1 + e_2 + e_3 = [4.88 - (1.464 \times 2)]10^{-4} = 1.952 \times 10^{-4}$$

$$V = 200 \times 20 \times 20 = 80000 \text{ mm}^3$$

$\therefore$ $\delta V = 1.952 \times 80000 \times 10^{-4} = 15.62 \text{ mm}^3$ **Answer**

Problem

A piece of steel 1 metre long and 3 cm x 3 cm cross section is subjected to a tensile force of 120 kN in the direction of its length. Calculate the change in volume. Take 1/m = 0.29 and E = 205 kN/mm^2.

Answer: 246 mm^3

Problem

A metal specimen is compressed in the direction of its axis and means are employed to reduce lateral expansion to one third of what it would be if free to expand. Calculate the modified value of elastic constant. Prove that its value will be (9/8)E if m = 4.

Answer: $E' = \dfrac{3m(m-1)}{3m^2 - 3m - 4}$

Example

A bar 30 mm in diameter was subjected to tensile load of 54 kN and the measured extension on 300 mm gauge length was 0.112 mm and change in diameter was 0.00366 mm. Calculate Poisson's ratio and the values of three moduli.

Solution

Stress = 76.4 N/mm^2

Linear strain = 0.112/300 = 3.73 x 10^{-4}

E = stress/strain = 204.6 kN/mm^2

Lateral strain = δd/d = 0.00366/30 = 1.22 x 10^{-4}

But lateral strain = (1/m)e = (1/m) (3.73 x 10^{-4})

$$\frac{1}{m}(3.73 \times 10^{-4}) = 1.22 \times 10^{-4}$$

∴ 1/m = 0.326.

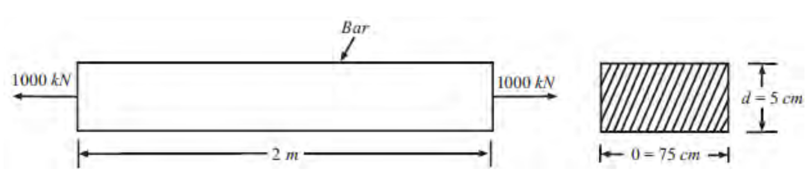
Again $$G = \frac{E}{2\left\{1 + \frac{1}{m}\right\}} = \frac{204.6}{2(1 + 0.326)} = 77.2 \text{ kN/mm}^2$$

$$K = \frac{E}{3\left\{1 - \frac{2}{m}\right\}} = 196 \text{ kN/mm}^2$$

where K = bulk modulus. G = Modulus of rigidity. **Answer**

Example

A 2m long rectangular bar of 7.5 cm × 5 cm is subjected to an axial tensile load of 1000kN. Bar gets elongated by 2mm in length and decreases in width by 10 × 10^{-6} m. Determine the modulus of elasticity E and Poisson's ratio of the material of bar.



Solution

Given:

$L = 2m$;

$B = 7.5cm = 0.075m$;

$D = 5cm = 0.05m$

$P = 1000kN$

$\delta L = 2mm = 0.002m$

$\delta b = 10 \times 10^{-6}$ m.

Longitudinal strain

$e_L = e_t = \delta L/L = 0.002/2 = 0.001$

Lateral strain

$\delta b/b = 10 \times 10^{-6}/0.075 = 0.000133$

Tensile stress (along the length)

$\sigma_t = P/A = (1000 \times 1000)/(0.075 \times 0.05) = 0.267 \times 10^9$ N/m^2

Modulus of elasticity,

$E = \sigma_t/e_t = 0.267 \times 10^9/0.001$ $\mathbf{= 267 \times 10^9}$ **N/m^2**

Poisson's ratio = Lateral strain/ Longitudinal strain

$= (\delta b/b)/(\delta L/L) = 0.000133/0.001$

Problem

A bar of elastic material is subjected to direct stress in a longitudinal direction, and its strain in the two directions at right angles are reduced to one half and one third respectively to those which normally occur in an ordinary tension member. If E = 200 kN/mm^2 and m = 4, what is the value of the elastic constant?

Answer: 221 kN/mm^2

Example (AMIE Summer 2015, 10 marks)

A 500 mm long bar has rectangular cross-section 20 mm x 40 mm. 1 lie bar is subjected to:

(i) 40 kN tensile force on 20 mm x 40 mm faces

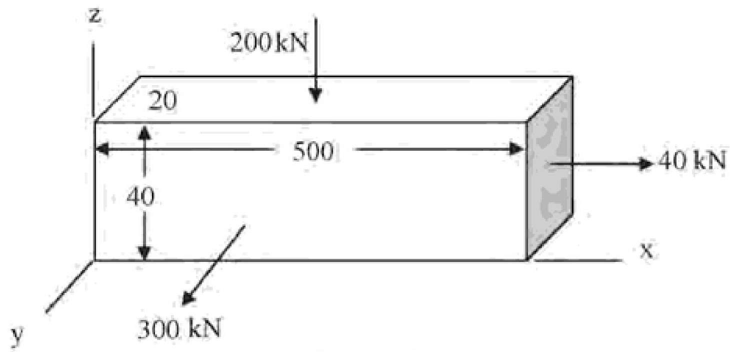
(ii) 200 kN compressive force on 20 mm x 500 mm faces

(iii) 300 kN tensile force on 40 mm x 500 mm faces

Find the changes in dimensions and volume, if E = 2 x 105 N/mm^2 and Poisson's ratio m = 0.3.

Solution

Figure



The dimensions of the bar and forces are shown in given figure.

Stresses

The stresses on various planes are:

$$\sigma_x = \frac{40x10^3}{20x40} = 50N/mm^2$$

$$\sigma_y = \frac{300x10^3}{500x40} = 15N/mm^2$$

$$\sigma_z = \frac{-200x10^3}{20x500} = -20N/m^2$$

Strains

$$e_x = \frac{1}{E}\left[\sigma_x - m(\sigma_y + \sigma_z)\right] = \frac{1}{2x10^5}\left[50 - 0.3(15 - 20)\right] = 2.575x10^{-4}$$

$$e_y = \frac{1}{E}\left[\sigma_y - m(\sigma_x + \sigma_z)\right] = \frac{1}{2x10^5}\left[15 - 0.3(50 - 20)\right] = 3x10^{-5}$$

$$e_x = \frac{1}{E}\left[\sigma_z - m(\sigma_x + \sigma_y)\right] = \frac{1}{2x10^5}\left[-20 - 0.3(50 + 15)\right] = -1.975x10^{-4}$$

$$e = e_x + e_y + e_z = 9x10^{-5}$$

Changes in dimension

Δx = 500 x e_x = 500 x 2.575 x 10^{-4} = 0.1288 mm

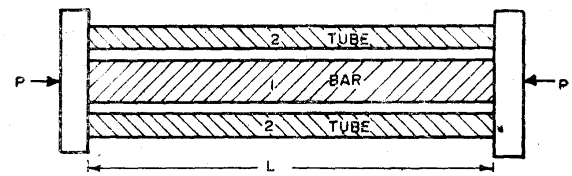
Δy = 20 x e_y = 20 x 3 x 10^{-5} = 6 x 10^{-4} mm

Δz = 40 x e_z = -40 x 1.975 x 10^{-4} = -7.9 x 10^{-3}

Change in volume

$$\Delta V = Ve = 4 \times 10^5 \times 9 \times 10^{-5} = 36 \text{ mm}^3$$

COMPOSITE SECTION



When a bar consists of two different materials, it is said to be composite. Since there ate two unknowns, two equations will be required. The conditions of equilibrium will provide one equation for the stresses in the individual sections. The other equation can be obtained from consideration of the deformation of the whole structure.

Let us take the case of solid bar enclosed in the hollow tube and subjected to a compressive force P through rigid collars as shown in figure. Using suffix 1 for the bar and 2 for the tube, we get from conditions of equilibrium

$$P_1 + P_2 = P \quad (1)$$

Since the whole assembly in composite, the deformation of bar is equal to the deformation of the tube. Thus

$$\Delta_1 = \Delta_2$$

or

$$\frac{P_1L}{A_1E_1} = \frac{P_2L}{A_2E_2} \quad (2)$$

Solving

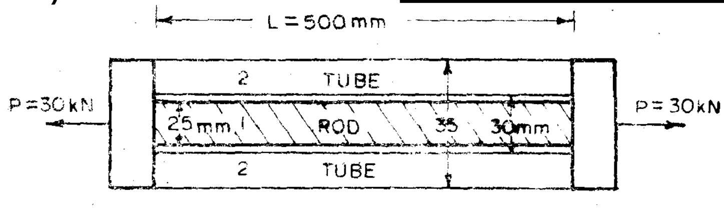
$$P_1 = \frac{P}{1 + \frac{A_2E_2}{A_1E_1}}$$

and

$$P_2 = \frac{P}{1 + \frac{A_1E_1}{A_2E_2}}$$

Example

A copper rod 25 mm in diameter is encased in steel tube 30 mm internal diameter and 35 mm external diameter. The ends are rigidly attached. The composite bar is 500 mm long and is subjected to an axial pull of 30 kN. Find the stresses induced in the rod and the tube. Take E for steel = 2 x 10^5 N/mm^2 and E for copper as 1 x 10^5 N/mm^2.



Solution

Let us use suffix 1 for rod (copper) and 2 for tube (steel)

$$A_1 = \frac{\pi}{4}(25)^2 = 490.9\,\text{mm}^2$$

$$A_2 = \frac{\pi}{4}\left[(35)^2 - (30)^2\right] = 255.3\,\text{mm}^2$$

Now $$p_1A_1 + p_2A_2 = P = 30000\text{N} \qquad (1)$$

also $$\frac{p_1L}{E_1} = \frac{p_2L}{E_2}$$

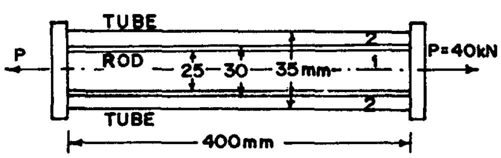
or $$p_1 = p_2\frac{E_1}{E_2} = \frac{2x10^5}{1x10^5}p_2 = 2p_2 \qquad (2)$$

From (1) $$2p_2(490.9) + p_2(255.3) = 30000$$

Solving p_2 = 24.25 N/mm^2 and $p_1 = 2p_2$ = 48.5 N/mm^2

Problem

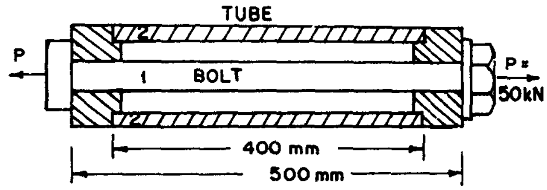
A copper rod 25 mm in diameter is encased in steel tube 30 mm internal diameter and 35 mm external diameter. The ends are rigidly attached. The composite bar is 400 mm long and is subjected to an axial pull of 40 kN. Find the stresses induced in the rod and the tube. Take E for steel = 2 x 10^5 N/mm^2 and E for copper as 1 x 10^5 N/mm^2.



Answer: steel: 64.68 N/mm^2; copper: 32.34 N/mm^2

Example

Two steel plugs fit freely into the ends of steel tubular distance piece 400 mm long and are drawn together by a steel bolt (500 mm long) and nut, the nut being tight fit in the beginning. The nut is further tightened by 1/4 turn to draw the pieces together, the pitch of the bolt thread being 2 mm. The pieces A and B are then subjected to forces of 50 kN tending to pull *them apart. Calculate the stresses in the bolt and the tube. The area of cross section of the bolt is 700 sq. mm and that of the tube is 500 sq. mm. E for steel = 200 GN/m^2.*



Solution

Let p_1 (tensile) and p_2 (compressive) be the final stresses in the bolt and tube respectively.

For equilibrium,

(tensile force in 1) - (compressive force in 2) = external tensile force

$\therefore \quad p_1A_1 - p_2A_2 = 50000$

or $\quad 700p_1 - 500p_2 = 50000$

or $\quad p_1 - 0.714p_2 = 71.43 \qquad (1)$

From compatibility,

(total extension of bolt) + (total compression of tube) = movement of nut

or $\quad \Delta_1 + \Delta_2 = \text{movement of nut}$

$$\therefore \quad \frac{p_1l_1}{E_1} + \frac{p_2l_2}{E_2} = \frac{1}{4}x2$$

$$\text{or} \quad p_1\frac{400}{2.05x10^5} + p_2\frac{500}{2.05x10^5} = 0.5$$

$$\text{or} \quad p_1 + \frac{2}{3}p_2 = \frac{0.5x2.05x10^5}{450} = 227.78 \qquad (2)$$

Solving (1) and (2)

p_1 = 138.62 N/mm^2 (tensile) and p_2 = 94.1 N/mm^2 (compressive)

Problem

Two steel plugs fit freely into the ends of steel tubular distance piece 300 mm long and are drawn together by a steel bolt (450 mm long) and nut, the nut being tight fit in the beginning. The nut is further tightened by 1/4 turn to draw the pieces together, the pitch of the bolt thread being 2 mm. The pieces A and B are then subjected to forces of 60 kN tending to pull them apart. Calculate the stresses in the bolt and the tube. The area of cross section of the bolt is 600 sq. mm and that of the tube is 400 sq. mm. E for steel = 205 GN/m^2.

Answer: p_1 = 163.9 N/mm^2 (tensile) and p_2 = 95.8 N/mm^2 (compressive)

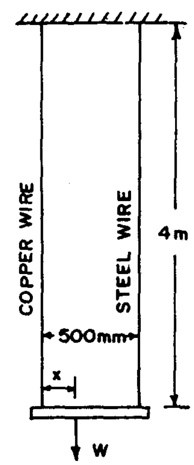
Example

Two vertical wires are suspended at a distance apart of 500 mm as shown in figure. Their upper ends are firmly secured and their lower ends support a rigid horizontal bar which carries load W. The left hand wire has a diameter of 1.6 mm and is made of copper and right hand wire has an area of 0.8 mm and is made of steel. Both wires initially are exactly 4 m long.

(a) Determine the position of the line of action of W, if due to W both wires extend by the same amount, and

(b) Determine the slope of wire if a load of 500 N is hung at the centre of the bar. Neglect the weight of bar and take E for steel = 200 kN/mm^2 and E for copper = 120 kN/mm^2.

Solution



(a) Let suffix s stand for steel bar and c for copper bar.

Let P_c and P_s be the forces in copper and steel wire respectively.

From statics $P_c + P_s = W$ (1)

Taking moment about steel wire

$$P_1 = \frac{W(500 - x)}{500} \quad (2)$$

Taking moment about copper wire

$$P_2 = \frac{Wx}{500} \quad (3)$$

Since AB remains horizontal

$$\Delta_c = \Delta_s$$

or $$\frac{P_c}{A_c E_c} = \frac{P_s}{A_s E_s} \quad (4)$$

$$\therefore \quad \frac{P_c}{P_s} = \frac{A_c}{A_s} \cdot \frac{E_c}{E_s} = \frac{2.011}{0.503} x \frac{120}{200} = 2.399$$

Also $\frac{P_c}{P_s} = \frac{500 - x}{x}$, from (2) and (3)

$$\therefore \quad \frac{500 - x}{x} = 2.399$$

i.e. x = 147.11 mm.

(b) Slope of wire if W is hung in the middle of bar

From (2)

$$P_s = P_c = \frac{W}{2} = 250N$$

Now $$\Delta_c = \frac{P_c l}{A_c E_c} = \frac{250\,x\,4000}{2.011\,x\,1.2\,x\,10^5} = 4.144 \text{ mm}$$

$$\Delta_s = \frac{P_s l}{A_s E_s} = \frac{250\,x\,4000}{0.503\,x\,2\,x\,10^5} = 9.940 \text{ mm}$$

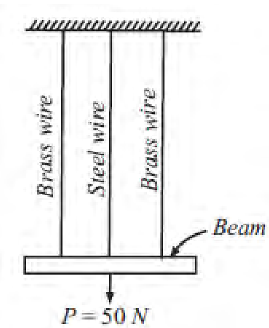
If the θ is the slope of the bar, we get

$$\tan\theta = \frac{9.940 - 4.144}{500} = 0.011592$$

∴ $\theta = 0.6641^0 = 0^0 39.85'$ (clockwise)

Example

A beam weighing 50 N is held in horizontal position by three wires. The outer wires are of brass of 1.8 mm dia and attached to each end of the beam. The central wire is of steel of 0.9 mm diameter and attached to the middle of the beam. The beam is rigid and the wires are of the same length and unstressed before the beam is attached. Determine the stress induced in each of the wire. Take Young's modulus for brass as 80 GN/m^2 and for steel as 200 GN/m^2.



Solution

If P_b denotes load taken by each brass wire and P_s denotes load taken by steel wire, then

Total load $P = 2P_b + P_s = 2\sigma_b A_b + \sigma_s A_s$ (i)

As the beam is horizontal, all the wire extend by the same amount. Further since each wire is of same length, the wires would experience the same amount of strain, thus

$e_s = e_b$

$\sigma_s/E_s = \sigma_b/E_b$

$\sigma_s = (E_s \times \sigma_b)/E_b = (200 \times \sigma_b)/80 = 2.5\sigma_b$ (ii)

Putting the value of equation (ii) in equation (i)

$50 = 2\sigma_b(\pi/4)(1.8)^2 + 2.5\sigma_b(\pi/4)(0.9)^2$

$50 = 6.678\ \sigma_b$

$\boldsymbol{\sigma_b = 7.49\ N/mm^2}$

$\boldsymbol{\sigma_s = 2.5\sigma_b = 18.71\ N/mm^2}$

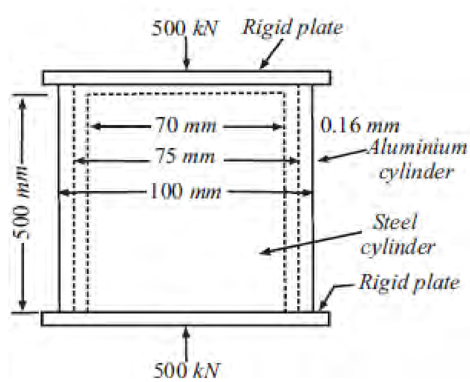
Problem

A load of 20 kN is jointly supported by the vertical rods equal in length, each 15 mm diameter, and equidistant in a vertical plane. Initially, the rods are so adjusted as to share the load equally. The outer rods are of steel and the middle one of copper. Calculate the total stresses if a further load of 15 kN is added. Take E_c = 110 kN/mm^2; E_s = 205 kN/mm^2.

Answer: 71.2 N/mm^2, 55.75 N/mm^2

Example (AMIE Winter 2008, 12 marks)

A solid steel cylinder 500 mm long and 70 mm diameter is placed inside an aluminium cylinder having 75 mm inside diameter and 100 mm outside diameter. The aluminium cylinder is 0.16 mm. longer than the steel cylinder. An axial load of 500kN is applied to the bar and cylinder through rigid cover plates as shown in Fig.. Find the stresses developed in the steel cylinder and aluminium tube. Assume for steel, E = 220 GN/m^2 and for Al E = 70 GN/m^2



Solution

Since the aluminium cylinder is 0.16 mm longer than the steel cylinder, the load required to compress this cylinder by 0.16 mm will be found as follows :

$$E = \text{stress/ strain} = P.L/A.\delta L$$

Or $$P = E.A.\delta L/L$$

$$= 70 \times 10^9 \times \pi/4\ (0.1^2 - 0.075^2) \times 0.00016/0.50016 = 76944\ N$$

When the aluminium cylinder is compressed by its extra length 0.16 mm, the load then shared by both aluminium as well as steel cylinder will be,

$$500000 - 76944 = 423056\ N$$

Let e_s = strain in steel cylinder

e_a = strain in aluminium cylinder

σ_s = stress produced in steel cylinder

σ_a = stress produced in aluminium cylinder

$E_s = 220\ GN/m^2$

$E_a = 70\ GN/m^2$

As both the cylinders are of the same length and are compressed by the same amount

$e_s = e_a$

$\sigma_s/E_s = \sigma_a/E_a$

or; $\sigma_s = E_s/E_a.\sigma_a = (220 \times 10^9/70 \times 10^9)$.

$\sigma_a = (22/7).\ \sigma_a$

Also $P_s + P_a = P$

or; $\sigma s\ A_s + \sigma_a \cdot A_a = 423056$

$(22/7).\ \sigma_a A_s + \sigma_a A_a = 423056$ ...(i)

$A_s = \pi/4\ (0.07^2) = 0.002199\ m^2$

$A_a = \pi/4\ (0.12 - 0.075^2) = 0.003436\ m^2$

Putting the value of A_s and A_a in equation (i) we get

$\sigma_a = 27.24 \times 10^6\ N/m^2$ **= 27.24 MN/m²**

and $\sigma_S = 22/7 \times 27.24$ = **85.61 MN/m²**

Stress in the aluminium cylinder due to load 76944 N

$= 76944/\ \pi/4\ (0.12 - 0.0752)$

$= 23.39 \times 10^9\ N/m^2 = 22.39\ MN/m^2$

Total stress in aluminium cylinder

= 27.24 + 22.39 = 49.63 MN/m²

and stress in steel cylinder **= 85.61 MN/m²**

Example

A concrete column 300mm x 300mm in section, is reinforced by 10 longitudinal 20mm diameter round steel bars. The column carries a compressive load of 450KN. Find load carried and compressive stress produced in the steel bars and concrete. Take $E_s = 200GN/m^2$ and $E_c = 15GN/m^2$.

Solution

Cross sectional Area of column = 300 × 300 = 90000mm^2

Area of steel bars As = 10 × (π/4)(20)^2 = 3141.59mm^2

Area of concrete = 90000 – 3141.59 = 86858.4 mm^2

Each component (concrete and steel bars) shorten by the same amount under the compressive load, and therefore

Strain in concrete = strain in steel

$\sigma_c/E_c = \sigma_s/E_s$

Where σ_c, σ_s are stress induced in concrete and steel respectively

$\sigma_s = (E_s\,\sigma_c)/E_c$

$= (200 \times 10^9/15 \times 10^9)\,\sigma_c = 13.33\,\sigma_c$

Further,

Total load on column = Load carried by steel + load carried by concrete

$P = \sigma_s.A_s + \sigma_c.A_c$

$450 \times 10^3 = 13.33\sigma_c \times 3141.59 + \sigma_c.\ 86858.4$

$= 128735.8\,\sigma_c$

$\boldsymbol{\sigma_c = 3.59\ N/mm^2}$

$\boldsymbol{\sigma_s = 13.33 \times 3.59 = 46.95\ N/mm^2}$

Load carried by concrete Pc = σ_{cAc} = 3.59 × 86858.4 **= 311821.656 N = 311.82KN**

Load carried by steel $P_s = \sigma_{sAs}$ = 46.95 × 3141.59 **= 147497.65 N = 147.49 KN**

Problem

A reinforced concrete column 500 mm diameter has four steel rods of 30 mm diameter embedded in it and carries a load of 680 kN. Find the stresses in steel and concrete. Take E for steel = 2.04 x 10^5 N/mm^2 and E for concrete = 0.136 x 10^5 N/mm^2. Find also the adhesive force between steel and concrete.

Answer: p_s = 43.24 N/mm^2, p_c = 2.88 N/mm^2, 112.475 kN

ASSIGNMENT

EQUILIBRIUM OF BODIES

Q.1. (AMIE S12, 4 marks): State what do you mean by equilibrium equation and compatibility condition.

Q.2. (AMIE S15, 18, W16, 4 marks): What is free body diagram? Write the purpose of free body diagrams. Discuss steps to construct free body diagrams with examples.

Q.3. (AMIE W17, 2 marks): State Lami's theorem.

Q.4. (AMIE S18, 4 marks): Write the statement of Varignon's theorem or principle of moments.

Answer: The theorem states that the moment of a resultant of two concurrent forces about any point is equal to the algebraic sum of the moments of its components about the same point. In other words, "If many coplanar forces are acting on a body, then the algebraic sum of moments of all the forces about a point in the plane of the forces is equal to the moment of their resultant about the same point."

Q.5. (AMIE W17, 2 marks): Define coefficient of friction.

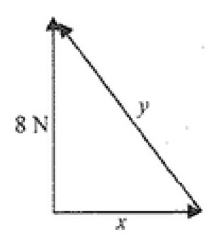
Answer: For qualitative definition Coefficient of friction is defined as the level of roughness or smoothness of a surface. Higher the coefficient of friction higher the roughness. The coefficient of friction value 0 indicates fully smooth surface.

For quantitative definition, its defined as the ratio of frictional force acting on between two surface to the normal reaction between two surface.

Q.6. (AMIE W17, 8 marks): Sum of magnitudes of forces in 16 N. The line of action of their resultant is perpendicular to the smaller force. Determine the magnitudes of these two forces if the magnitude of the resultant is 8 N and the angle of indication between the two forces.

Answer: 6 N and 10 N

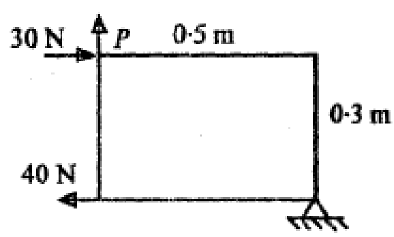
Hint: See figure.



Q.7. (AMIE W17, 8 marks): A thin form rod AB is hinged at lower end A and the upper end B is attached by a horizontal string of length 4 m from a point C, 1.5 m vertically above B. If the weight of the rod is 5 N find the tension in the horizontal string.

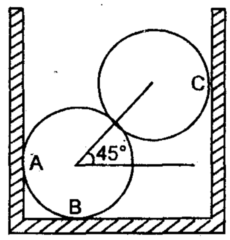
Answer: 13.33 N

Q.8. (AMIE S18, 10 marks): A rectangular plate is held in equilibrium by application of forces shown in Fig. Determine the magnitude of unknown force P and reaction at the hinge. Neglect the weight of plate.



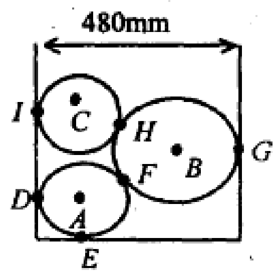
Answer: 19.4365 N

Q.9. (AMIE W05, 10 marks): Two equal cylinders, each weighing 900 N are placed in a box as shown in figure. Neglecting friction between the cylinders and the box, estimate the reactions at A, B and C.



Answer: 900 N, 1800 N, 900 N

Q.10. (AMIE S07, 11, 14 marks): Three cylinders *A*, *B*, *C* of weights 100N, 200N, 300 N and radii 100 mm, 200 mm, 150 mm are placed in a rectangular ditch. Neglecting friction, determine reactions at various contact points.

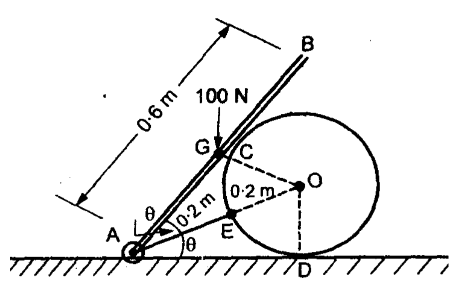


Answer: R_G = -1.35 N, R_D = 89.69 N, R_F = 139.48 N, R_H = 242.9 N, R_I = -91 N

Q.11. (AMIE S12, 10 marks): Cylindrical roller A, weighing 1500 kg having diameter 50 cm, and another cylinder roller By weighing 1200 kg having diameter 42 cm, are kept in a horizontal channel of width 72 cm. Curved surfaced touches the base of the channel and one vertical face of the channel, and curved surface of B also touches the other vertical face of the channel. Assuming no friction, find the reaction at all the contact points.

Answer: 822.35 Kg, 822.35 Kg and 2700 Kg

Q.12. (AMIE W06, 10 marks): The figure below shows a smooth cylinder of radius 0.2 m supporting a rod AB 0.6 m long weighing 100 N. The end A of the rod is hinged to the horizontal surface AD. The cylinder is also attached to the hinge by a string of length 0.2 m. Find the tension in the string.



Answer: 113.394 N

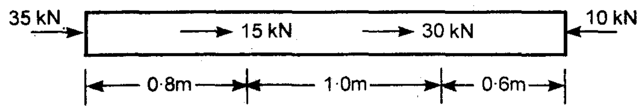
SIMPLE STRESSES & STRAINS

Q.13. (AMIE W15, S11, 13, 17, 3 marks): Write the expressions for generalised Hook's law.

Q.14. (AMIE W17, 6 marks): What do you mean by plain stress and plain strain condition?

Q.15. (AMIE W11, 5 marks): Express stresses σ_x, σ_y and σ_z in terms of strains ε_x, ε_y and ε_z using generalised Hook's law.

Q.16. (AMIE S05, 10 marks): The aluminium bar with a cross section area of 160 mm^2 carries the axial loads at the positions shown in following figure.



Given that E = 70 GPa, compute total change in length of the bar.

Answer: 4.227 mm

Q.17. (AMIE S08, 13, 8 marks): A mild steel bar, 6 m long, is 5 cm in diameter for 3 m of its length and 2.5 cm in diameter for the remaining length. The bar is in tension and the stress on the smallest section is 112 MPa. Find the total elongation of the bar and the change in diameter at the smallest section. Given: E = 200 GPa and Poisson's ratio = 0.15.

Answer: 0.21 cm, - 0.021 cm

Q.18. (AMIE S08, 13, 8 marks): A rod of square section of side D at one end tapers to a square section at the other end. If its length is l, find the increase in length if it is subjected to an axial pull P.

Answer: Pl/EDd

Q.1. (AMIE S15, 6 marks): A bar of length 50 cm has varying cross-section. It carries a load of 25 kN. Find the extension, if the cross- section is given by [5 + (x^2/100)] cm^2, where x is the distance from one end in cm. Neglect weight of the bar. Take E = 2 x 10^{11} N/m^2.

Answer: 0.004 cm

Hint: $\Delta = \int_0^{50} d\Delta = \int_0^{50} \frac{Pdx}{\left[5 + \frac{x^2}{100}\right](2x10^7)}$

Q.19. (AMIE S17, 8 marks): Find an expression for elongation of a bar of rectangular section and a conical section due to self weight.

Answer: WL/2AE

ELASTIC CONSTANTS

Q.1. (AMIE S11, 17, W10, 16, 17, 5 marks): Define Poisson's ratio. Write the expressions for modulus of rigidity (G) and bulk modulus (K) in terms of Young's modulus of elasticity (E) and Poisson's ratio (μ). Establish the relation between E, K and G.

Q.2. (AMIE S17, 6 marks): Define bulk modulus. Deduce the relation

$$E = 3K (1 - 2\mu)$$

Q.20. (AMIE W06, 10 marks): Define (i) Modulus of elasticity (ii) Modulus of rigidity (iii) Bulk modulus and establish the relationship among them.

Q.1. (AMIE W11, 16, 6 marks): What is volumetric stress and volumetric strain? How is the volumetric strain determined when a circular rod is loaded axially?

Q.2. (AMIE S06, 8 marks): A specimen of 15 mm diameter and 200 mm long is subjected to tensile test and data at proportional and elastic limits were recorded as below:

Limit	Stress (MPa)	Increase in length (mm)	Reduction in diameter (mm)
Proportional	340	00.90	?
Elastic	350	01.00	0.0225

Find modulus of elasticity, Poisson's ratio and reduction in diameter.

Answer: 70 GPa, 0.3, 0.02025 mm

Hint: E = stress/strain = 350 x 10^6/0.005 = 70 GPa

Q.21. (AMIE W05, 06, 16, S06, 10, 6 marks): What is generalised Hook's law? Write the governing equations for two dimensional stet of stress and three dimensional state of stress.

Q.22. (AMIE S06, 4 marks): For two dimensional stress system, show that

$$\sigma_1 = \frac{E}{1-v^2}\left(\varepsilon_1 + v\varepsilon_2\right)$$

and

$$\sigma_2 = \frac{E}{1-v^2}\left(v\varepsilon_1 + \varepsilon_2\right)$$

where v is Poisson ratio.

Q.23. (AMIE W08, 10 marks): A bar of 20 mm diameter is subjected to an axial tensile load of 120 kN, under which 200 mm gauge length of this bar elongates by an amount of 3.5 x 10^{-4} m. Determine the modulus of elasticity of the bar material. if μ = 0.3, determine its change in diameter.

Answer: 0.0105 mm

Q.24. (AMIE S09, 8 marks): A bar 30 mm x 30 mm x 250 mm long is subjected to a pull of 90 kN in the direction of its length. The extension of the bar was found to be 0.125 mm, while the decrease in each lateral dimension is found to be 0.00375 mm. Find the Young's modulus, Poisson's ratio, modulus of rigidity and bulk modulus for the material of the bar.

Answer: E = 20 x 10^4 N/mm^2, Poisson's ratio = 0.25, G = 8 x 10^4 N/mm^2, K = 13.34 x 10^4 N/mm^2

Q.1. (AMIE S16, 6 marks): A cube of material is under the action of a uniform compressive stress on its faces. If the volume of the cube decreases by 0.1%, find the uniform stress on the cube. Also, find the percentage reduction in length of a side of the cube. Take E = 2 x 10^5 N/mm^2 and v = 0.3.

Answer: σ = 167 N/mm^2; 0.033%

Q.2. (AMIE S17, 8 marks): A bar of 24 mm diameter and 400 mm length is acted upon by an axial load of 38 kN. The elongation of the bar and the change in diameter are 0.165 mm and 0.0031 mm respectively. Determine (i) Poisson's ratio (ii) values of the three moduli.

Answer: m = 0.313; E = 203636 MPa; G = 77546 MPa; K = 181494 mPa

COMPOSITE SECTION

Q.25. (AMIE S06, 10 marks): A structural member 5 m long is made up of two materials. The first 1.7 m of its length is of brass and is 7.5 cm^2 in cross section and the remainder of its length is of steel and is 6.0 cm^2 in cross section. The bar is in tension under load P Newton and the total elongation of the bar is 0.1 cm. Determine (i) magnitude of load (ii) work done in elongation of the bar. Take E_s = 210 GPa and E_b = 84 GPa.

Answer: 18.8 kN, 940 N-cm

Q.26. (AMIE W12, 10 marks): A copper sleeve, 21 mm internal and 27 mm external diameter, surrounds a 20 mm steel bolt, one end of the sleeve being in contact with the shoulder of the bolt. The sleeve is 60 mm long. After putting a rigid washer on the other end of the sleeve, a nut is screwed on the bolt through 10°. If the pitch of the threads is 2.5 mm, find the stresses induced in the copper sleeve and steel bolt. Take E = 200 GN/m^2 and E = 90 GN/m^2.

Answer: σ_c = 0.078 N/mm^2, σ_s = 0.056 N/mm^2

Hint: Contraction induced due to turning nut through 10^0 = (10/360) x 2.5 = 2.5/36 mm

Contraction of copper sleeve + extension of the steel bolt = 2.5/36 mm

Q.27. (AMIE W11, 8 marks): Two rods A and B of equal free length hang vertically 60 cm apart and support a rigid bar horizontally. The bar remains horizontal when carrying a load of 5000 kg at 20 cm from rod A . If the stress in B is 50 N/mm^2, find the stress in rod A and the areas of A and B. Take E_A - 200,000 N/mm^2 and E_B = 90,000 N/mm^2.

Answer: 111.11 N/mm^2, A_B = 327 mm^2, A_A = 294.3 mm^2

Q.28. (AMIE W09, 10 marks): An aluminium solid cylinder of 7.5 cm diameter fits loosely inside a steel tube having 10 cm external diameter and 8 cm internal diameter. The steel tube is 0.02 cm longer than aluminium cylinder and is 250 cm long before the load is applied. Calculate the safe load which can be placed on a rigid flat plate on the lop of the steel tube. Safe stress for steel is 95 MPa and for aluminium 65 MPa, E_s = 210 GPa, and E_{al} = 70GPa.

Answer: 408.83 kN

Q.29. (AMIE W11, 8 marks): A 75 mm diameter compound bar is constructed by shrinking a circular brass bush on to the outside of a 50 mm diameter solid steel rod. If the compound bar is then subjected to an axial compressive load of 160 kN, determine the load carried by the steel rod and brass bush and the compressive stresses set up in each material. Take E_{steel} =210,000 N/mm^2 and E_{brass} = 90,000 N/mm^2.

Answer: σ_b = 34.82 N/mm^2, σ_s = 81.48 N/mm^2

Q.30. (AMIE W09, 10 marks): A load of 300 kN is applied on a shoo concrete column 250 mm x 250 mm. The column is reinforced by steel bars of total area 5600mm^2 If modulus of elasticity of steel is 15 limes that of concrete, find the stresses in concrete and steel. If this stress in concrete should not exceed 4N/mm^2. find the area of steel required so dial the column may support a load of 600kN.

Answer: σ_s = 60 N/mm^2, σ_c = 31.95 N/mm^2, A_s = 6250 mm^2

Q.31. (AMIE W15, 10 marks): A copper rod of 40 nun diameter is surrounded tightly by a cast iron tube of 80 mm external diameter, the ends being firmly fastened together. When put to a compressive load of 30.000 N, what load will be shared by each? Also determine the amount by which the compound bar shortens if it is 2000 mm long?

Answer: 3750 N, 26.250 N, 0.0796 mm

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